Robust Helicopter Position Control at Hover

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Abstract
Position control for a radio controlled helicopter at hover is carried out using $H_2$ and $H_{\infty}$ methods. Various problems arising in real-world tests are stated and solved by different means: prefilters are used to achieve step responses without overshoot. For plants with more outputs than inputs, a special $H_{\infty}$ design scheme is used. The implicit inversion of the plant in the $H_{\infty}$ controller can cause problems. This inversion can be avoided by a new scheme for weighting the sensitivity.

1 Introduction
Helicopter control is a rather typical example for multivariable robust control applications since the dynamics of rotorcraft are highly coupled and nonlinear. To really check the quality of any robust controller design, it is necessary to verify the system behavior in real-world tests rather than just in simulations. For several years, our laboratory has been using an indoor test bench for rotorcraft hover control. Its configuration is described in detail in [5]. A certain number of control theories have already been applied and tested in the past (e.g., LQG/LTR design, modal controller design). These evaluations have led to some adaptations of the algorithms as shown in [5]. The results of recent tests of $H_2$ and $H_{\infty}$ controller designs are presented in this paper.

Section 2 contains a short description of the model as well as the goals and limitations of the controller design. The configuration and results of $H_2$ design are briefly presented in Section 3, while Section 4 is dedicated to $H_{\infty}$ design. A comparison of the different strategies is given in Section 5.

2 Plant description

2.1 Modelling the helicopter
In human-piloted helicopters, attitude or rate controllers are typically used. For our model helicopter, we want to design autonomous controllers, i.e., we want to control the inertial position and the yaw angle of the rotorcraft. Twelve state variables are necessary to describe the rigid-body motion of the fuselage. Two state variables are added for the first-order flapping motion of the auxiliary rotor attached to the main rotor (Bell-Hiller system). The dynamics of the driving DC motor require two additional state variables.

Finally, for this small rotorcraft, the delayed reaction of the induced velocity to sudden pitch changes must be respected; it is approximated by a first-order system for each rotor. Hence, the physics of our helicopter are given by a model of 18 states. A state-space description of this plant is given in [5].

The multiplexing radio controller used for control signal transmission induces a time delay of 80 ms in each control channel. For linear controller designs, these delays are taken into account by second-order Padé approximations.

This extended linearized system with 26 states is unstable and has two poles in the right half-plane and four poles at the origin. With four zeros having a positive real part, it is nonminimum phase.

This model consists of two parts with different physical backgrounds: A first subsystem describes vertical and yaw motions which are controlled by the collective angles of attack of the main and tail rotors, respectively. They are strongly coupled by the torque interaction between the main rotor and the fuselage. On the other hand, forward and sideward motions, together with pitch and roll motions, are controlled by longitudinal and lateral cyclic stick inputs via the flapping motion of the main rotor (second subsystem).

Since in hover conditions, the interaction of these two parts of the helicopter is weak, it is possible to design controllers for these two subsystems separately. Although a controller design for the whole helicopter is possible too, it entails some additional problems, as described in Section 4.3.

In the second subsystem, it is necessary to measure roll and pitch angles in addition to forward and sideward position because without these measurements, the plant would have two pairs of transmission zeros on the imaginary axis. Such zeros would prevent any robust controller design, [3].

2.2 Constraints and goals for the controller design
The main aim of the controller design is to permit hovering as smoothly as possible even in disturbed air. This implies that we need a good disturbance rejection, thus a small sensitivity function in the low frequency range. Additionally, the bandwidth should be as high as possible.

![Figure 2.1 Common closed-loop system.](image)

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The following constraints restricting the design must be taken into account. Experiments have shown that the closed-loop transfer function

$$T_{vr} = G_p K (1 + G_p K)^{-1}$$

must remain below $-15$ dB at the frequency $\omega = 50$ rad/s in order to avoid exciting structural modes. Stability robustness for nonlinear and unmodelled plant uncertainties shall be achieved by limiting both the sensitivity and the complementary sensitivity to 6 dB. Since it is not possible to shape a loop anywhere but at a location where the signal vector is of minimum size equaling the rank of the closed-loop transfer function, the controller for the second subsystem must be designed at the plant input. Consequently, the design limitations are given at the plant input. The first subsystem is square and well-conditioned. Hence, the constraints can be applied either at the input or at the output of the plant with similar results.

### 3 $H_2$ controller designs

In statically weighted $H_2$ design (LQG/LTR) as described in [5], bandwidth and low frequency gains are interdependent. With dynamic weighting, however, low frequency gains do not depend on the bandwidth. Using the augmentation scheme of Figure 3.1, the following weightings lead to the best results:

First subsystem:

$$W_s = \begin{bmatrix} s^2 + 2s + 1 \\ 2s^2 + 2s + 5 \\ 0 \\ \sqrt{3} s^2 + 4s + 4 \\ 2s^3 + 4s + 2 \end{bmatrix},$$

$$W_d = \sqrt{18} I_2, \quad W_r = I_2, \quad W_d = 10^{-2} I_2.$$

Second subsystem:

$$W_s = \begin{bmatrix} s + 8 \\ 4s + 2 \\ 0 \\ \sqrt{3} (4s^2 + 4s + 1) \end{bmatrix},$$

$$W_d = \sqrt{0.03} \frac{s + 28}{\sqrt{2}(s + 14)} I_2, \quad W_r = I_2, \quad W_d = 10^{-2} I_2.$$

Without changing robustness margin and bandwidth, the static gain is increased by a factor of 2.1 for the first subsystem (vertical and yaw motion), and by a factor of 3.6 for the second subsystem, compared to the LQG/LTR design in [5].

The closed-loop transfer function for unstable plants always exceeds 0 dB for certain frequencies (cf. Figs. 3.2 and 3.3), as shown in [3]. Thus, step responses tend to overshoot, and a feedforward controller design as described in [1] is applied for both subsystems to avoid this behavior.

Figures 3.4 and 3.5 show test bench measurements of a vertical- and a forward-step response, respectively. Notice the good quality of decoupling in the vertical step response. The yaw angle remains constant within ±3 degrees (.05 rad). The restless behavior at the end of these measurements is due to the air disturbances in the closed room where the tests are conducted.

![Figure 3.1](image)

**Figure 3.1** $H_2$ augmentation scheme.

![Figure 3.2](image)

**Figure 3.2** Sensitivity and complementary sensitivity for the first subsystem's $H_2$ design.

![Figure 3.3](image)

**Figure 3.3** Sensitivity and complementary sensitivity for the second subsystem's $H_2$ design.

![Figure 3.4](image)

**Figure 3.4** Vertical-step response with $H_2$ controller.
4.1 Design for vertical and yaw motion
The first subsystem is square, i.e., the number \( m \) of inputs is equal to the number \( p \) of outputs. This allows the use of the common \( \mathcal{H}_\infty \) design scheme (Figure 4.1) to achieve the design goals stated in Section 2.2.

As in Section 3, the closed-loop transfer function cannot be designed without any resonance. Thus, the two-degree-of-freedom (2DOF) scheme of [1] is applied for good reference tracking (Figure 4.2).

With the following weightings, the static gain is 5% higher than in the \( \mathcal{H}_2 \) case, thus slightly increasing the quality of disturbance rejection.

\[
W_c = \begin{bmatrix}
\frac{45}{s^2 + 11.136s + 31} & 0 \\
0 & \frac{45}{s^2 + .103s + 3.10^{-4}}
\end{bmatrix},
\]

\[
W_e = \begin{bmatrix}
\frac{s^3}{s^3 + 3.8} & 0 \\
0 & \frac{17680}{s + 4.4}
\end{bmatrix},
\]

The singular value plots shown in Figure 4.3 look similar to the ones of Figure 3.2. The maximum resonance of the complementary sensitivity is even reduced by a factor of .6.

4.2 Design for forward and sideward motions
For the second subsystem, the common \( \mathcal{H}_\infty \) design scheme of Figure 4.1 cannot be applied since there are more measurement signals than control signals. The scheme of Figure 4.5 can be used instead. Here, the loop is designed at the plant input, i.e., the sensitivity \( S_p \) at the plant input is shaped with the weight \( W_e \), while \( W_c \) defines the comple-
mentary sensitivity $T_w$ at the same location. With this configuration, well-shaped frequency responses are achieved, and the simulation results look similar to those of the $H_2$ case.

![Figure 4.5 $H_\infty$ weighting scheme for design at the plant input.](image)

In a practical application, however, a remarkable drawback can be observed: the plant contains two resonances, one at .1 and one .17 rad/s. They are compensated by this $H_\infty$ controller such that the controller ends up with poles at .04 and .09 rad/s (Figure 4.6). These controller modes are excited after the helicopter takes off, causing it to oscillate slowly for several minutes.

![Figure 4.6 Singular values of the plant $G_p$ and the controller $K$ designed with the scheme of Figure 4.5.](image)

One attempt to avoid this behavior would be to copy the resonances of the plant into the weighting $W_e$. Since the eigenvectors of these modes are coupling the system, this would be rather difficult to do, however.

The same goal can be reached by using the design scheme of Figure 4.7. Here, the sensitivity $S_w$ is shaped by the weight $W_eG_p$, thus the plant resonances are no longer compensated. Clearly, the design configuration shown in Figure 4.7 for the $H_\infty$ approach is equal to that of Figure 3.1 for the $H_2$ design.

Eventually, the tuning with this layout leads to the following weightings:

$W_f = \begin{bmatrix} \frac{9}{s^2} & \frac{s^2}{s^2} & 64 + \frac{6}{s^2} & \frac{1}{s^2} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} .07943I_2 \\ 0 \end{bmatrix}$

$W_e = \frac{26.5}{(s^2 + 15 + \sqrt{2} + 125)(s + 13)} I_2,$

$W_r = \begin{bmatrix} .01I_2 & 0 \\ 0 & .01I_2 \end{bmatrix}.$

The steady-state gain can be increased by 25% over the value in the $H_2$ design. In analogy to Figure 4.2, a one-step prefilter design can be realized with the help of the configuration depicted in Figure 4.8.

![Figure 4.7 Modified $H_\infty$ design scheme for the second subsystem.](image)

The additional weightings are as follows:

$W_M = \frac{1}{s^3 + 15.12s^2 + 72.576s + 110.592} I_2,$

$W_r = \frac{1.6923}{s^2 + 200s + 100^2} I_2.$

This results in a step response with negligible overshooting as shown in Figure 4.9.

![Figure 4.9 Forward-step response with $H_\infty$ controller.](image)

### 4.3 $H_\infty$ controller design for the complete helicopter

After designing a controller for the first subsystem with the configuration of Figure 4.7, it should be possible to do a design for the complete helicopter by connecting the weights of the subsystems. However, the $H_\infty$ algorithm does not converge for this case. The reason for this is rooted in the following: since the outputs $z$ and the inputs $w$ of the augmented plant are connected by an unstructured uncertainty of the norm $\rho \gamma$ (i.e., $||\Delta||_\rho \leq \rho \gamma$; where $\gamma$ is the achieved $H_\infty$ norm of $T_{zw}$), with a larger number of signals
in \( z \) and \( w \), the unstructured uncertainty has more possibilities for destabilizing the system.

The goal of \( \gamma = 1 \) can be reached by halving the crossover frequency of \( W_c \). The consequence of that reduction, however, is an unsatisfactorily sluggish behavior in real flight as depicted in Figure 4.10.

For all three designs, the peak values of the off-diagonal elements of the closed-loop transfer function from the reference input to the output remain below -20 dB. Hence, input-output coupling is negligible compared to the un-modelled plant dynamics.

Static \( H_2 \) design (LQG/LTR) with guaranteed robustness is easy to use and allows for a quick first estimate of the possible bandwidth. With the \( H_\infty \) design, a remarkably higher performance is reached. This improvement, however, can only be achieved if the actual constraints and disturbances of the plant are well known. Thus, the \( H_\infty \) design allows for better results, but it requires a great deal of additional knowledge. For the design goals stated here, the dynamic \( H_2 \) design requires the same knowledge of the plant as the \( H_\infty \) design, but it cannot reach the same level of performance.

Thus, if the quality reached with LQG/LTR is sufficient, stop there, otherwise \( H_\infty \) represents the method of choice.

5 Comparison

The comparison of the various controllers is based on the design objectives of Section 2.2 on the one hand; on the other hand, the complexity of the algorithms are to be taken into account as well.

All of the designs presented go to the limit of the robustness constraints, thus, static gain and bandwidth may serve as measures of performance quality. In Figure 5.1, these characteristics are illustrated, indicating that \( H_\infty \) performs best.

References


