Techniques are described for carrier frequency offset (CFO) and channel estimation of orthogonal frequency division multiplexing (OFDM) transmissions over multiple-input multiple-output (MIMO) frequency-selective fading channels. A wireless transmitter forms blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols. The transmitter applies a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols, and a modulator outputs a wireless signal in accordance with the blocks of symbols. A receiver receives the wireless signal and estimates the CFO, and outputs a stream of estimated symbols based on the estimated CFO.

30 Claims, 12 Drawing Sheets
References Cited

U.S. PATENT DOCUMENTS

6,850,481 B2 2/2005 Wu et al.
6,865,175 B1 3/2005 Ritter
6,959,010 B1 10/2005 Bahenbush et al.
7,069,931 B2 3/2006 Ma et al.
7,065,371 B1 6/2005 Kleiman
7,161,973 B1 1/2007 Ghosh
8,064,528 B2 11/2011 Giannakis et al.
2004/0001429 A1 1/2004 Ma et al.
2004/019636 A1 9/2004 Oprea

OTHER PUBLICATIONS

}
References Cited


ERIC-1001
Ericsson V. RUM
Page 3 of 28
References Cited

OTHER PUBLICATIONS


References Cited

OTHER PUBLICATIONS


ETSI, "Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical (PHY) layer; ETSI TS 101 475 V1.2.2, 2001, 41 pages.


FIG. 1
FIG. 2
FIG. 3
TRANSMIT

INSERT \( N \) TRAINING SYMBOLS ACROSS \( M \) SPACE-TIME ENCODED BLOCKS OF INFORMATION-BEARING SYMBOLS

APPLY HOPPING CODE TO INSERT \( N - K \) NULL SUBCARRIERS PER BLOCK WITH POSITION HOPPING

OUTPUT OFDM TRANSMISSION SIGNAL BY INSERTING A CYCLIC PREFIX AND TAKING THE IFFT OF THE BLOCK

RECEIVE

RECEIVE THE OFDM TRANSMISSION SIGNAL AND REMOVE THE CYCLIC PREFIX

APPLY DE-HOPPING CODE AND ESTIMATE THE CFO

TAKE FFT AND REMOVE NULL SUBCARRIERS

PERFORM CHANNEL ESTIMATION USING \( M \) BLOCKS OF TRAINING SYMBOLS

ESTIMATE AND REMOVE THE PHASE NOISE

DECODE THE INFORMATION-BEARING SYMBOLS TO PRODUCE SYMBOL ESTIMATES

FIG. 4
FIG. 10
FIG. 11

AVERAGE CHANNEL NMSE vs. SNR
ESTIMATING FREQUENCY-OFFSETS AND MULTI-ANTENNA CHANNELS IN MIMO OFDM SYSTEMS

This invention relates to communication systems and, more particularly, carrier frequency offset estimation and channel estimation in communication systems.

BACKGROUND

Orthogonal frequency division multiplexing (OFDM) is a paramount goal in developing coding and modulation schemes. When a data rate for wireless communication systems is high in relation to bandwidth, multipath propagation may become frequency-selective and cause intersymbol interference (ISI). Multipath fading in wireless communication channels causes performance degradation and constitutes the bottleneck for increasing data rates.

OFDM has been adopted by many standards because it offers high data-rates and low decoding complexity. For example, OFDM has been adopted as a standard for digital audio broadcasting (DAB) and digital video broadcasting (DVB) in Europe and high-speed digital subscriber lines (DSL) in the United States. OFDM has also been proposed for local area mobile wireless broadband standards including IEEE 802.11a, IEEE 802.11g, MMAC and HIPERLAN/2. Additionally, space-time (ST) multiplexing with multiple antenna arrays at both the transmitter and receiver are effective in mitigating fading and enhancing data rates. Therefore, multi-input multi-output (MIMO) OFDM is inherently resistant to multipath fading and has been adopted by many standards because it offers high data-rates and low decoding complexity.

OFDM systems rely on blocks of training symbols and unique sequences, which are transmitted to the receiver. Unlike conventional systems in which training symbols are sent at a known position, in OFDM systems training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block and are decoupled from symbol detection at the receiver. Unlike conventional systems in which training symbols are sent at a known position, in OFDM systems training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block and are decoupled from symbol detection at the receiver. Unlike conventional systems in which training symbols are sent at a known position, in OFDM systems training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block and are decoupled from symbol detection at the receiver.

Unlike conventional systems in which training symbols are sent at a known position, in OFDM systems training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block and are decoupled from symbol detection at the receiver. Unlike conventional systems in which training symbols are sent at a known position, in OFDM systems training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block and are decoupled from symbol detection at the receiver.

In general, the invention is directed to techniques for carrier frequency offset estimation and channel estimation of orthogonal frequency division multiplexing (OFDM) transmissions over multiple-input multiple-output (MIMO) frequency-selective fading channels. In particular, techniques are described that utilize training symbols such that CFO and channel estimation are decoupled from symbol detection at the receiver. Unlike conventional systems in which training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block, the techniques described herein insert training symbols over two or more transmission blocks. Furthermore, training symbols may include both non-zero symbols and zero symbols and are inserted such that the training symbols and channel symbols are received in a format in which the training symbols are easily separated from the information-bearing symbols, thereby enabling carrier frequency estimation to be performed prior to channel estimation.

One embodiment of the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols.

In one embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols.

In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols.

In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols.

In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols. In another embodiment, the invention is directed to a method comprising forming blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; applying a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and outputting wireless transmission signal in accordance with the blocks of symbols.
ing symbols, and at least one null subcarrier at a different position within each of the blocks of symbols; an carrier frequency offset estimator to estimate a carrier frequency offset of the received signal based on the positions of the null subcarriers; and a decoder to output a stream of estimated symbols based on the received wireless signal and the estimated carrier frequency offset.

In another embodiment, the invention is directed to a computer-readable medium containing instructions. The instructions cause a programmable processor to form blocks of symbols by inserting training symbols within two or more blocks of information-bearing symbols; apply a hopping code to each of the blocks of symbols to insert a null subcarrier at a different position within each of the blocks of symbols; and output wireless transmission signal in accordance with the blocks of symbols.

The described techniques may offer one or more advantages. For example, instead of performing CFO and MIMO channel estimation on a per block basis, several transmission blocks are collected by a receiver for estimating CFO and the MIMO frequency-selective channels, thereby resulting in an efficient use of bandwidth. Further, because the training symbols are inserted in a manner that decouples CFO and channel estimation from symbol detection, low-complexity CFO and channel estimation can be performed. Moreover, the described techniques allow for full acquisition range of the CFO estimator and identifiability of the MIMO channel estimator.

Other advantages of performing block equalization may include improved bit-error-rate (BER) performance relative to typical techniques and flexibility to adjust the number of blocks collected to perform channel estimation. Because of the improved BER performance, less expensive voltage controlled oscillators may be used. Additionally, the training patterns of the described techniques can easily be implemented by current OFDM standards, such as IEEE 802.11a and IEEE 802.11g.

The details of one or more embodiment of the invention are set forth in the accompanying drawings and the description below. Other features, objects, and advantages of the invention will be apparent from the description and drawings, and from the claims.

**BRIEF DESCRIPTION OF DRAWINGS**

FIG. 1 is a block diagram illustrating an exemplary wireless multi-user communication system in which multiple transmitters communicate with multiple receivers through a wireless communication channel.

FIG. 2 is a block diagram illustrating in further detail one embodiment of a transmitter and a receiver within the multi-user communication system of FIG. 1.

FIG. 3 illustrates example transmission blocks generated by the transmitter of FIG. 2.

FIG. 4 is a flowchart illustrating an example mode of operation of the communication system of FIG. 2 in which a receiver performs CFO estimation and channel estimation on an OFDM transmission signal output by a transmitter.

FIGS. 5-12 are graphs illustrating performance estimates of the CFO and channel estimation techniques described herein.

**DETAILED DESCRIPTION**

Throughout the Detailed Description bold upper letters denote matrices, bold lower letters stand for column vectors, ( )' denotes transpose and Hermitian transpose, respectively; ( )* denotes conjugate and [•] denotes the near-est integer. E[•] stands for expectation and diag[•] stands for a diagonal matrix with x on its main diagonal; matrix D_x(h) with a vector argument denotes an N×N diagonal matrix with D_x(h)diag[h]. For a vector, ||[•]|| denotes the Euclidian norm. A_{m,n} denotes the (k, m)th entry of a matrix A, and x_{k,n} denotes the nth entry of the column vector x. e_{l} denotes the N×N identity matrix; e denotes the (i+1)st column of I_N; [F_{x,m,n}]_{N×N} denotes the fast fourier transform (FFT) matrix; and we define f_{1}, f_{2}, ..., f_{N-1} as:

\[ f_k = 1, \exp(j\omega), ..., \exp(j(N-1)\omega). \]

**FIG. 1** is a block diagram illustrating a multi-user wireless communication system 2 in which multiple transmitters communicate with multiple receivers 6 through wireless communication channel 8. In general, the invention describes techniques for performing carrier frequency offset (CFO) and channel estimation of orthogonal frequency division multiplexing (OFDM) transmissions output by transmitters 4 over multiple-input multiple-output (MIMO) frequency-selective fading channel 8. As described herein, the techniques maintain orthogonality among subcarriers of OFDM transmissions through channel 8 allowing low-complexity receivers 6 and full acquisition range of the CFO.

Transmitters 4 output a transmission signal in accordance with a block of symbols in which two or more training symbols are inserted and in which a hopping code is applied. A block of training symbols including two or more training symbols may be inserted within a block of space-time encoded information-bearing symbols. A hopping code may then be applied to the resulting block of symbols to insert a null subcarrier, i.e. zero symbol, within the block symbols such that the null subcarrier changes position, i.e. “hops”, from block to block. Unlike conventional systems in which training symbols are inserted within a single transmission block, the techniques described herein insert training symbols over two or more transmission blocks. Consequently, transmitters 4 may insert a sequence of training symbols over two or more transmission blocks, thereby increasing bandwidth efficiency because smaller blocks of training symbols may be used. Receivers 6 may then collect the training symbols inserted within the two or more transmission blocks in order to perform channel estimation. Furthermore, the information-bearing symbols and training symbols are received through communication channel 8 by receivers 6 in a format in which the training symbols are easily separated from the information-bearing symbols, thereby enabling CFO estimation to be performed prior to channel estimation. As a result, the techniques described herein may have improved bit-error-rate (BER) performance over conventional alternatives.

The described techniques can work with any space-time encoded transmission and is backwards compatible with OFDM which has been adopted as a standard for digital audio broadcasting (DAB) and digital video broadcasting (DVB) in Europe and high-speed digital subscriber lines (DSL) in the United States. OFDM has also been proposed for local area mobile wireless broadband standards including IEEE 802.11a, IEEE 802.11g, MMAC and HIPERLAN/2.

The techniques described herein apply to uplink and downlink transmissions, i.e., transmissions from a base station to a mobile device and vice versa. Transmitters 4 and receivers 6 may be any device configured to communicate using a multi-user wireless transmission including a cellular distribution station, a hub for a wireless local area network, a cellular phone, a laptop or handheld computing device, a personal digital assistant (PDA), a Bluetooth enabled device, and other devices.

**FIG. 2** is a block diagram illustrating in further detail the multi-user communication system of FIG. 1. In particular,
FIG. 2 illustrates exemplary embodiments of multi-antenna transmitter 4 and multi-antenna receiver 6 communicating over MIMO frequency-selective channel 8 in the presence of a CFO. Multi-antenna transmitter 4 and multi-antenna receiver 6 have Nt and Nt antennas, respectively. While OFDM transmissions are inherently resilient to multipath fading, OFDM transmissions are more sensitive to frequency offsets than single carrier systems. Frequency offsets can occur when a voltage controlled oscillator (VCO) of receiver 6 is not oscillating at exactly the same carrier frequency as a VCO of transmitter 4 and can also occur as a result of the Doppler effect. When the frequency offset is permanent, it is typically referred to as a carrier frequency offset and when the frequency offset varies over time, it is typically referred to as phase noise. Frequency offsets cause a degradation in BER performance because the orthogonality among subcarriers is destroyed and the subcarriers can interfere with each other.

Generally, receiver 6 corresponds to a particular user performing CFO and channel estimation of OFDM transmissions output by transmitter 4 through MIMO frequency-selective fading channel 8 in the presence of a CFO. Each information-bearing symbol s(n) 10 is selected from a finite alphabet and the input to serial to parallel converter (S/P) 11 which parses Nt information-bearing symbols from a serial stream of information-bearing symbols into blocks of information-bearing symbols. The nth entry of the kth block of the block of information-bearing symbols is denoted [s(k)],=s (kn+n) (kn+n) for each block s(k) in space and time to yield blocks {cµ(k)}µ~IN, 14 of length Nt. Space-Time coder 13 encodes and/or multiplexes each block s(k) in space-time to yield blocks {cµ(k)}µ~IN, 14 for each respective transmit antenna of multi-antenna transmitter 4.

Each of training symbol insertion units 15 inserts two or more training symbols, which may have non-zero or zero values, within a space-time encoded block {cµ(k)}µ~IN, 14 and applies a hopping code to blocks {cµ(k)}µ~IN, 14 to form a vector uµ(k) 16 with length Nt for the µth antenna of multi-antenna transmitter 4. Applying the hopping code inserts a null subcarrier which changes position, i.e. "hops", from block to block. Each subcarrier corresponding to a zero symbol is referred to as a null subcarrier. Unlike conventional systems in which training symbols are inserted within a single transmission block, each of training symbol insertion units 15 may insert training symbols over two or more blocks. Consequently, transmitter 4 may insert a sequence of training symbols over two or more blocks {cµ(k)}µ~IN, 14. Sparsely inserting training symbols increases the bandwidth efficiency of communication system 8 because fewer training symbols may be inserted per block {cµ(k)}µ~IN, Nt. In some embodiments, each of training symbol insertion units 15 may insert a particular number of training symbols per block {cµ(k)}µ~IN, Nt based on channel 8’s coherence time and the pertinent burst duration, e.g. if the burst is long fewer training symbols may be inserted per block {cµ(k)}µ~IN, 14. Furthermore, training symbols may be inserted in accordance with existing OFDM standards such as IEEE 802.11a and IEEE 802.11g. Training symbol insertion units 15 are described in greater detail below using notation introduced in the following paragraphs.

Subsequent to the insertion of training symbols, MIMO OFDM is implemented. In particular, each of inverse fast Fourier transform (IFFT) units 17 implement N-point IFFT via left multiplication with F16 on each corresponding block uµ(k) 16 and each of cyclic prefix insertion units 19 insert a cyclic prefix via left multiplication with the appropriate matrix operator T16=1-16×16,F16×16, where I16×16 represents the last L columns of I16. Each of parallel to serial converters (PS) 21 then parses the resulting blocks {uµ(k)=1-16×16F16×16dµ(k)}µ~IN, of size P×1 into a serial symbol stream. Each of modulators 23 modulate the corresponding P×1 blocks which are transmitted by the Nt transmit antennas over frequency-selective communication channel 8.

Consequently, communication channel 8 can be represented in the discrete-time equivalent form hν(µ) (l)={gν *gν}ν=0...L, where ν denotes convolution and T is the sampling period which may be chosen to be equivalent to the symbol period.

Transmissions over communication channel 8 experience a frequency offset fν in Hertz, which may be caused by a mismatch between a voltage controlled oscillator (VCO) of transmitter 4 and a VCO of receiver 6 or may also be caused by the Doppler effect. In the presence of a frequency offset, the samples at with receive antenna can be represented according to equation (1) below, where ων=2πfνT is the normalized CFO, Nt is the number of receive antennas, and gν(n) is zero-mean, white, complex Gaussian distributed noise with variance σν2.

Each of serial to parallel converters (S/P) 25 convert a received sequence s(n) into a corresponding P×1 block 26 with entries [s(k)]=:x(k)=s(k)+p). By selecting block size P greater than channel order L each received block s(k) 26 depends only on two consecutive transmitted blocks, uµ(k) and uν(k-1) which is referred to as inter-block interference (IBI). In order to substantially eliminate IBI at receiver 6, each of cyclic prefix removers 27 removes the cyclic prefix of the corresponding blocks s(k) 26 by left multiplication with the matrix R=100...01. The resulting IBI-free block can be represented as yµ(k):=Rν,s(k) 28. Equation (2) below can be used to represent the vector-matrix input-output relationship, where yµ(k)=[yµ(kP), yµ(kP+1), . . . , yµ(kP+P-1)T]T, with P=NL, Hν(µ) is a P×P lower triangular Toeplitz matrix with first column [hν(µ),hν(µ)+1,...,hν(µ)+P-1], and Dν(ων) is a diagonal matrix defined as Dν(ων)=diag{1, eνωνT,...,eνωνT}.
to simplify the input-output relationship, \( F \) may be one block of training symbols in a training sequence. Moreover, \( b_{\mu}(k) \) the presence of CFO \( \omega \) estimation can be separated from MIMO channel by multiplying \( \tilde{u}_{\mu}(k) \) with the hopping code \( T_{sc}(k) \), where \( \tilde{u}_{\mu}(k) \) is formed by permuting the entries of \( h^{(\mu)}(k) \) as dictated by \( T_{sc}(k) \). Using the de-hopping code given in equation (10), the identity given in equation (11) can be established, where \( T_{sc}(k)=[I_{2N}, 0_{2N} \cdots 0_{2N}] \) is a zero-padding operator.

By multiplying equation (9) by the de-hopping code and using equation (11), equation (12) is obtained, where
Proposition 1 If \( \mathbb{E}[b_{\mu}(k)b_{\mu}^H(k)] \) is diagonal, \( \mathbb{E}[c_{\mu}(k)c_{\mu}^H(k)] = 0 \), and \( \mathbb{E}[b_{\mu 1}(k)b_{\mu 2}^H(k)] = 0 \), then

\[
\sum_{\mu=1}^{N} \mathbb{E}[b_{\mu}(k)b_{\mu}^H(k)]
\]
has full rank, \( \mathbb{E}[c_{\mu}(k)c_{\mu}^H(k)] = 0 \), and \( \mathbb{E}[b_{\mu 1}(k)b_{\mu 2}^H(k)] = 0 \), \( \forall \mu_1, \mu_2 \), then

\[
\sum_{\mu=1}^{N} \mathbb{E}[c_{\mu}(k)c_{\mu}^H(k)]
\]
has full rank.

Training block \( b_{\mu}(k) \) satisfies the conditions of proposition 1. Using the result of Proposition 1,

\[
\sum_{\mu=1}^{N} \mathbb{E}[c_{\mu}(k)c_{\mu}^H(k)]
\]
has full rank.

The column space of \( R_y \) has two parts: the signal subspace and the null subspace. In the absence of CFO, if \( \mathbb{E}[g(k)g^H(k)] \) has full rank, the null space of \( R_y \) is spanned by missing columns, i.e., the location of the null subcarriers, of the FFT matrix. However, the presence of CFO introduces a shift in the null space. Consequently, a cost function can be built to measure this CFO-induced phase shift. Representing the candidate CFO as \( \omega_0 \), this cost function can be written according to equation (15), where

\[
J(\omega) = \sum_{\mu=1}^{N} \frac{1}{N} \mathbb{E}[g(k)g^H(k)] T_{\mu} F_{\mu} R_y F_{\mu} D_{\mu}(\omega_0) + \mathbb{E}[g(k)g^H(k)]
\]

Consequently, if \( \omega = \omega_0 \), then \( D_{\mu}(\omega_0, \omega) = 1 \). Next, recall that the matrix \( F_{\mu}^H T_{\mu} \) is orthogonal to \( \{f_{\mu}(2\pi m, N)\}_{m=1}^{N} \). Therefore, if \( \omega = \omega_0 \), the cost function \( J(\omega_0) \) is zero in the absence of noise. However, for this to be true, \( \omega_0 \) must be the unique minimum of \( J(\omega) \). \( \omega_0 \) is the unique zero of \( J(\omega) \) if

\[
\sum_{\mu=1}^{N} \mathbb{E}[g(k)g^H(k)]
\]
has full rank as established in Proposition 1 below.

The column space of \( R_y \) has two parts: the signal subspace and the null subspace. In the absence of CFO, if \( \mathbb{E}[g(k)g^H(k)] \) has full rank, the null space of \( R_y \) is spanned by missing columns, i.e., the location of the null subcarriers, of the FFT matrix. However, the presence of CFO introduces a shift in the null space. Consequently, a cost function can be built to measure this CFO-induced phase shift. Representing the candidate CFO as \( \omega_0 \), this cost function can be written according to equation (15), where

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\[
\sum_{\mu=1}^{N} \mathbb{E}[g(k)g^H(k)]
\]
has full rank as established in Proposition 1 below.
Rh is typically unknown, thus, MN 6 ;;,;N,(L+l), and BHB is selected to have full rank. In some embodiments, channel estimation unit 33 is a least squares (LS) estimator given according to equation (25).

\[ \tilde{h}_{c}(k) = \tilde{h}_{c}(k) = \sum_{p=1}^{P} D_{h}(P_{h}(k)p_{T}(k) + \xi_{c}(k)) \]

By collecting \( z_{c}(k)'s \) from all N receive antennas into \( \Xi_{c} = \{ \ldots, \Xi_{c}(0) \ldots \} \), the linear MMSE (LMMSE) channel estimator can be expressed according to equation (24), where \( R_{c} = E[\bar{h}^{H}\bar{h}] + \sigma_{n}^{2} I \) as the channel covariance matrix, and \( \sigma_{n}^{2} \) represents the noise variance.

\[ \hat{h}_{MMSE} = (R_{c}^{-1} + \sigma_{n}^{2} B^{H}B)^{-1} R_{c}^{-1} + \sigma_{n}^{2} B^{H} \]

If the number of training symbols per block is \( N_{r} = N_{c} \), a minimum number of \( M = L+1 \) blocks are required to be collected by receiver 6 in order to guarantee that LS estimation can be performed since \( h^{(v,\mu)} \) with \( L+1 \) entries are estimated at the vth receive antenna. In some embodiments, channel estimation unit 33 can be adjusted to collect a variable number of blocks based on the complexity that can be afforded.

Further, assume \( N \) and \( M \) are integer multiples of \( L+1 \). Because the hopping step size in equation (8) is \( N/(L+1) \), BHB can be designed according to equation (27).

\[ B_{1}^{H}m_{0} = \sum_{m=0}^{M-1} P_{m}^{H} F_{m}(m) F_{m}^{H}(m) \]

By collecting \( z_{c}(k)'s \) from all N receive antennas into \( \Xi_{c} = \{ \ldots, \Xi_{c}(0) \ldots \} \), the linear MMSE (LMMSE) channel estimator can be expressed according to equation (24), where \( R_{c} = E[\bar{h}^{H}\bar{h}] + \sigma_{n}^{2} I \) as the channel covariance matrix, and \( \sigma_{n}^{2} \) represents the noise variance.

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If the number of training symbols per block is \( N_{r} = N_{c} \), a minimum number of \( M = L+1 \) blocks are required to be collected by receiver 6 in order to guarantee that LS estimation can be performed since \( h^{(v,\mu)} \) with \( L+1 \) entries are estimated at the vth receive antenna. In some embodiments, channel estimation unit 33 can be adjusted to collect a variable number of blocks based on the complexity that can be afforded.

Further, assume \( N \) and \( M \) are integer multiples of \( L+1 \). Because the hopping step size in equation (8) is \( N/(L+1) \), BHB can be designed according to equation (27).

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\[ \hat{h}_{MMSE} = (R_{c}^{-1} + \sigma_{n}^{2} B^{H}B)^{-1} R_{c}^{-1} + \sigma_{n}^{2} B^{H} \]

If the number of training symbols per block is \( N_{r} = N_{c} \), a minimum number of \( M = L+1 \) blocks are required to be collected by receiver 6 in order to guarantee that LS estimation can be performed since \( h^{(v,\mu)} \) with \( L+1 \) entries are estimated at the vth receive antenna. In some embodiments, channel estimation unit 33 can be adjusted to collect a variable number of blocks based on the complexity that can be afforded.
estimate the phase noise per block. For example, assume that for the kth block, the estimated channel is obtained by using the LMMSE channel estimator given in equation (24). Further, also assume that the training sequence is designed as given in equation (26) and that channel estimation is perfect, i.e., D(k) = δ(k). As a result, after equalizing channel 8, for the vth receive antenna and the µth entry of xv(k) 30, the equivalent input-output relationship is given according to equation (29), where vµ(k) = \frac{xv(k)}{\|h_{v\mu}(k)\|} 30, and \( w_v \) is the equivalent noise term after removing the channel.

\[ \phi_v(k) = \Phi_{v\mu}(k)D(k) + w_v \]  

(29)

Because \( \Phi_{v\mu}(k) \) is known, the phase \( \phi_v(k) = \Phi_{v\mu}(k) \) can be estimated based on the observations from \( N_r \) receive antennas on a per block basis. In order to perform this phase estimation step, additional training symbols do not need to be inserted and the extra complexity is negligible. The performance improvement resulting from phase estimation is illustrated in the performance graphs given below.

After CFO estimation, the FFT has been performed, and channel estimation space-time decoder 37 decodes the space-time encoded information-bearing symbols to produce the information-bearing symbol estimates \( \hat{s}_v(k) \) as described herein. In some embodiments, the number of training symbols inserted may be adjusted depending on the channel’s coherence time and the pertinent burst duration. Additionally, transmitter 4 may insert two or more training symbols per block of space-time encoded information-bearing symbols by applying a first and a second permutation matrix, \( P_d \) and \( P_{d2} \), respectively, as described previously. After inserting the training symbols, transmitter 4 applies a hopping code to insert \( N-K \) null subcarriers per block such that the position of the null subcarriers changes from block to block (step 52). The hopping code may be defined as in equation (8) with hop-step \( N/(L+1) \). It may be particularly advantageous to insert null subcarriers in accordance with conventional OFDM standards such as IEEE 802.11a and IEEE 802.11g. Transmitter 4 then outputs an OFDM transmission signal after inserting a cyclic prefix and taking the IFFT of the resulting block of training and information-bearing symbols (step 54).

Receiver 6 receives the OFDM transmission signal and removes the cyclic prefix (step 56). Receiver 6 then applies a de-hopping code and estimates the CFO (step 58). The de-hopping code reorders the null subcarriers so that the null subcarriers in different blocks are at the same position in their respective blocks, and the CFO is estimated as described previously. Because of the null subcarrier hopping, the CFO estimation and channel estimation can be separated and the CFO can be estimated over the full acquisition range \([-\pi, \pi] \).

The FFT is taken and the null subcarriers are removed (step 60) by multiplying \( \hat{s}_v(k) \) by zero padding matrix \( P_{d2} \) to obtain \( \hat{s}_v(k) \). Channel estimation is performed over \( M \) blocks of training symbols (step 62). As described previously, each training block length \( N_t \) can be smaller than \( N_r(L+1) \) by sparsely distributing training symbols across \( M \) blocks. In some embodiments, one of a LMMSE channel estimator or a LS channel estimator may be applied to the \( M \) blocks to estimate channel 8. In order to improve the BER performance of receiver 6, the phase noise is estimated and removed (step 64) based on the observations from \( N_r \) receive antennas on a per block basis. Symbol estimates are then produced by decoding the space-time encoded information-bearing symbols (step 66).

FIGS. 5-12 are graphs that present simulations of OFDM transmissions using the described techniques for estimating the CFO, channel, and phase noise. In order to benchmark the performance of the techniques described herein, the Cramer-Rao lower bounds (CRLB) for the CFO are derived. Starting from the model of communication system 2 given in equation (12), the CRLB for \( \omega \) is given according to equation (30), where \( D(k) = \text{diag}(P_{d1}, \ldots, P_{d(M+1)}) \), and \( R_{\hat{\omega}} = \text{E}[(\hat{\omega}(k) - \omega)^2] \).

\[ \text{CRLB}_\omega = \left( \sum_{k=1}^{N_t} \sum_{i=1}^{M} \frac{1}{\sigma_{i}^2} \text{E}[(\hat{\omega}(k) - \omega)^2] \right)^{-1} \]  

(30)

It follows from equation (30) that as the number of blocks increases, the CRLB for CFO decrease. Similarly, the signal-
to-noise ratio (SNR) versus CRLB decreases as the number of blocks increases. If $N_{M}=N-K$, i.e. the number of subcarriers is much greater than the number of null subcarriers, $T_{P}=T_{0}$.

Assuming that $R_{0}^{-1}, e_{0}$, where $e$ represents the average symbol energy, and $P_{M}$ are sufficiently large, equation (31) can be obtained:

$$CLRB_{w} = \frac{\sigma^{2}}{\epsilon} \cdot \frac{3}{N_{M}M^{N}} \cdot \frac{1}{N_{P}} \cdot \frac{1}{N_{T}}$$

Equation (31) explicitly shows that the CRLB of the CFO is independent of the channel and the number of transmit antennas, and that the CRLB of the CFO is inversely proportional to the SNR, the number of receive antennas, and the cube of the number of space-time data.

By assuming that CFO estimation is perfect, the performance of the channel estimator can be derived. If the LMMSE channel estimator given in equation (24) is used, then the mean-square error of the channel estimator is given according to equation (32). 

$$\sigma_{\text{error}}^{2} = \epsilon \left[ \left( \frac{1}{N_{P}} + \frac{M}{N_{T}} \right) a_{0}^{2} \right]$$

Similarly, if the LS channel estimator given in equation (25) is used, the corresponding mean-square error is given by equation (33).

$$\sigma_{\text{error}}^{2} = \frac{N_{S}N_{L}+1}{M}\sigma_{N}^{2}$$

Equations (32) and (33) both imply that as the number of channels increases, the channel mean square error increases. However, this increase can be mitigated by collecting a greater number of blocks, i.e. more training symbols, provided that the CFO estimate is sufficiently accurate.

In all simulations, HIPERLAN/2 channel model $B_{v}$ given in Table 1, is used to generate the channels. The channel order is $L=15$ and the taps are independent with different variances. The OFDM block length is designed as $N=64$ as in RIPER-2. The noise is additive white Gaussian noise with zero-mean and variance $\sigma_{N}^{2}$. The SNR is defined $SNR=E_{a}/N_{0}$.

FIG. 6 is a graph comparing the effect of the number of blocks on CFO estimation. The CFO is randomly selected in the range $[-0.5\pi, 0.5\pi]$. In each OFDM transmission block, there are four non-zero training symbols, 4 zero symbols to remove interference from other channels, and one zero symbol serving as a null subcarrier. The placement of the training symbols is in accordance with the techniques herein, and different numbers of blocks are used: $M=4$ (plot 80), $M=K$ (plot 82), $M=2K$ (plot 84), $M=3K$ (plot 86), and the CRLB derived previously with $M=K$ (plot 88) for comparison. FIG. 6 depicts the CFO error probability of mean square error (MSE), defined as $E[|\hat{\omega}_{0} - \omega_{0}|^{2}]$, versus SNR. As the number of OFDM blocks $M$ increases, the MSE of CFO decreases. However, the improvement is relatively small, which suggests that using $M=K$ OFDM blocks is sufficient to estimate the CFO.

FIG. 7 is a graph comparing the effect of the number of antennas on CFO estimation using the LS channel estimator given in equation (25). The average MSE of the CFO with the number of blocks $M=4$ are plotted as lines $90$, $92$, $94$, and $96$ for systems having $(N_{S}, N_{L})=(1, 1)$, $(N_{S}, N_{L})=(1, 2)$, $(N_{S}, N_{L})=(2, 1)$, $(N_{S}, N_{L})=(2, 2)$, respectively. For plots $90$ and $92$, one non-zero symbol and one null subcarrier per OFDM transmission block are used. FIG. 7 illustrates that as the number of receive antennas increases, the performance of the CFO estimation techniques described herein increases due to the receive-diversity gains.

TABLE 1

<table>
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<th>tap no.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
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<td>1.8e-02</td>
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</table>

<table>
<thead>
<tr>
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<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
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<td>6.5e-03</td>
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<td>2.5e-03</td>
<td>0.0e-03</td>
<td>1.4e-03</td>
<td>0.0e-03</td>
<td>6.0e-04</td>
</tr>
</tbody>
</table>

FIG. 5 is a graph comparing the true frequency offset versus the estimated CFO for the CFO estimation techniques described herein (plot 70) and an algorithm described in P. H. Morelli, "A technique for orthogonal frequency division multiplexing frequency offset correction," IEEE Transactions on Communications, vol. 42, pp. 2908-1314, October 1994 (plot 72). The ideal line (74) is also shown for comparison and illustrates that the currently described CFO estimation technique (plot 70) has the full acquisition range $[-\pi, \pi]$, whereas the algorithm described in the P. H. Morelli’s reference (plot 72) has an acquisition range proportional to the OFDM block size $N$.

FIG. 8 is a graph comparing the CFO estimation techniques described herein with a technique described in M. Morelli and U. Mengali, "An improved frequency offset estimator for OFDM applications," IEEE Communications Letters, vol. 3, pp. 75-77, March 1999, for the single antenna case. For the case of $M=4$, one non-zero training symbol and one zero training symbol for each block are used for each OFDM transmission block and 64 blocks are collected to perform CFO estimation. In order to maintain the same transmission rate, M. Morelli’s and U. Mengali’s previously referenced technique has a training block length of 128 with 8 identical parts. FIG. 8 depicts two cases: random CFO in $[-0.06\pi, 0.06\pi]$ and fixed CFO with $\omega_{0}=-1/128$. In both cases the CFO is chosen within the acquisition range of M. Morelli and U. Mengali’s previously referenced technique. In both cases, the CFO techniques described herein, 100 and 102 for the fixed CFO case and the varying CFO case, respec-
tively, are comparable with M. Morelli and U. Mengali’s technique for the fixed CFO case and varying CFO case.

Fig. 9 is a graph comparing the performance of MIMO channel estimation with $(N, N_r) = (2, 2)$ and the CFO being randomly selected in the range $[-0.5, 0.5]$. By collecting 64 observations from 8 OFDM transmission blocks and using the LS channel estimator given in equation (25), the MIMO channels can be estimated. In order to measure the channel estimation quality, the average channel NMSE is computed as $E[\|\mathbf{h} - \mathbf{h}_l\|^2_2]$, where $\mathbf{h}_l$ is obtained using the LS method. The performance for MIMO OFDM transmissions with estimated CFO 110 using the techniques described herein are compared with the ideal case in which the CFO is perfectly known 112. Fig. 9 illustrates a 4.5 dB loss due to the CFO estimation error.

Fig. 10 compares the BER performance of the CFO and channel estimation techniques described herein without phase noise estimation (plot 120), with phase noise estimation (plot 122), and with perfect phase noise estimation (plot 124) with increasing SNR. The simulation parameters are the same as those used in Fig. 9 and zero-forcing equalization is used to estimate the information-bearing symbols. The BER performance of all the simulations degrades as the number of blocks increase due to phase noise. As expected, the plot with phase noise estimation 122 performs better than the plot without phase noise estimation 120 and the plot with perfect phase noise estimation 124 provides a benchmark.

Figs. 11 and 12 compare the estimation of $N_c$ COFs in multiuser broadcast or OFDM systems. Simulations are performed with $(N, N_r) = (2, 2)$ and COFs are randomly selected in the range $[-0.5, 0.5]$. In particular, Fig. 11 illustrates the average channel NMSE with varying SNRs using a $N_c \times 1$ vector CFO estimator for the presently described techniques with $M=K$ (plot 130), $M=2K$ (plot 132), $M=3K$ (plot 134), $M=4K$ (plot 136). Similarly, Fig. 12 illustrates the BER performance with varying SNRs using the presently described CFO and channel estimation techniques without phase noise estimation (plot 140), with phase noise estimation (plot 142), and with perfect phase noise estimation (plot 144). Figs. 11 and 12 illustrate results which corroborate with Figs. 9 and 10 respectively. Consequently, the described techniques which were illustrated in detail for a single-user system involving $N_t$ transmit antennas and $N_r$ receive antennas, can be applied with similar results in a multi-user downlink scenario where the base station deploys $N_t$ transmit antennas to broadcast OFDM based transmissions to $N_r$ mobile stations each of which is equipped with one or more antennas.

Various embodiments of the invention have been described. The invention provides techniques for carrier frequency offset (CFO) and channel estimation of orthogonal frequency division multiplexing (OFDM) transmissions over multiple-input multiple-output (MIMO) frequency-selective fading channels. In particular, techniques are described that utilize training symbols in a manner that CFO and channel estimation are decoupled from symbol detection at the receiver. Unlike conventional systems in which training symbols are inserted within a block of space-time encoded information-bearing symbols to form a transmission block, the techniques described herein insert training symbols over two or more transmission blocks.

The described techniques can be embodied in a variety of transmitters and receivers used in downlink operation including cell phones, laptop computers, handheld computing devices, personal digital assistants (PDA’s), and other devices. The devices may include a digital signal processor (DSP), field programmable gate array (FPGA), application specific integrated circuit (ASIC) or similar hardware, firmware and/or software for implementing the techniques. If implemented in software, a computer readable medium may store computer readable instructions, i.e., program code, that can be executed by a processor or DSP to carry out one or more of the techniques described above. For example, the computer readable medium may comprise random access memory (RAM), read-only memory (ROM), non-volatile random access memory (NV-RAM), electrically erasable programmable read-only memory (EEPROM), flash memory, or the like. The computer readable medium may comprise computer-readable instructions that when executed in a wireless communication device, cause the wireless communication device to carry out one or more of the techniques described herein. These and other embodiments are within the scope of the following claims.

The invention claimed is:

1. A method comprising: in a base station: forming two or more blocks of output symbols for orthogonal frequency division multiplexing (OFDM) transmissions over a multiple-input multiple-output (MIMO) channel, wherein the forming comprises: (i) identifying different positions within the two or more blocks of output symbols based at least on (a) one or more block index values, (b) one or more subcarrier index values, (c) a cyclic prefix parameter selected to compensate for intersymbol interference (ISI) associated with the MIMO channel, and (d) a hopping code that is based, at least in part, on the cyclic prefix parameter, and (ii) inserting, based at least on the hopping code, training symbols and null subcarriers within two or more blocks of information-bearing symbols, the hopping code directing at least a portion of the null subcarriers to be inserted at the different positions within the two or more blocks of output symbols; and transmitting, via two or more antennas, transmission signals in accordance with the two or more blocks of output symbols, wherein the two or more blocks of output symbols include a first block of output symbols and a second block of output symbols; and wherein inserting the training symbols and null subcarriers comprises: inserting a first null subcarrier at a first subcarrier position within the first block of output symbols; and inserting a second null subcarrier at a second subcarrier position within the second block of output symbols, wherein the first subcarrier position is different from the second subcarrier position.

2. The method of claim 1, wherein transmitting the transmission signals comprises:

transmitting, via a first antenna of the two or more antennas, a first transmission signal in accordance with the first block of output symbols; and

transmitting, via a second antenna of the two or more antennas, a second transmission signal in accordance with the second block of output symbols.

3. The method of claim 1, wherein transmitting the transmission signals comprises inserting a cyclic prefix within each of the blocks of output symbols, and wherein the transmission signals provide information for estimating a carrier frequency offset associated with received versions of the transmission signals.

4. The method of claim 3, wherein the transmission signals provide information for estimating a phase noise of the received versions of the transmission signals based on the estimated carrier frequency offset.
6. The method of claim 1, wherein inserting the training symbols and the null subcarriers comprises inserting at least one training symbol adjacent to at least one null subcarrier.

7. The method of claim 1, wherein transmitting the transmission signals comprises transmitting transmission signals for a multi-user wireless communications system.

8. The method of claim 1, further comprising: encoding the information-bearing symbols in space and time within the two or more blocks of output symbols.

9. A system, comprising:
   a base station configured to (i) form two or more blocks of output symbols for orthogonal frequency division multiplexing (OFDM) transmissions over a multiple-input multiple-output (MIMO) channel by identifying different positions within the two or more blocks of output symbols based at least on (a) one or more block index values, (b) one or more subcarrier index values, (c) a cyclic prefix parameter selected to compensate for intersymbol interference (ISI) associated with the MIMO channel, and (d) a hopping code that is based, at least in part, on the cyclic prefix parameter, and inserting, based at least on the hopping code, training symbols and null subcarriers within two or more blocks of information-bearing symbols, the hopping code directing at least a portion of the null subcarriers to be inserted at the different positions within the two or more blocks of output symbols, and (ii) transmit, via the two or more antennas, transmission signals in accordance with the two or more blocks of output symbols, wherein the two or more blocks of output symbols include a first block of output symbols and a second block of output symbols, and wherein the base station is further configured to:
   - insert a first null subcarrier at a first subcarrier position within the first block of output symbols; and
   - insert a second null subcarrier at a second subcarrier position within the second block of output symbols, wherein the first subcarrier position is different from the second subcarrier position.

10. The system of claim 9, further comprising: a wireless communication device configured to receive the transmission signals as received signals, wherein the wireless communication device is configured to (i) estimate a carrier frequency offset based on the received signals and (ii) perform channel estimation of the MIMO channel based on the received signals.

11. The system of claim 10, wherein the wireless communication device is configured to estimate a phase noise of the received signals based on the carrier frequency offset.

12. The system of claim 9, wherein the base station is configured to:
   - transmit, via a first antenna of the two or more antennas, a first transmission signal in accordance with the first block of output symbols; and
   - transmit, via a second antenna of the two or more antennas, a second transmission signal in accordance with the second block of output symbols.

13. The system of claim 9, wherein the base station is configured to insert a cyclic prefix within each of the blocks of output symbols, and wherein the transmission signals provide information for estimating a carrier frequency offset associated with received versions of the transmission signals.

14. The system of claim 9, wherein the training symbols within the two or more blocks collectively provide information for estimating the MIMO channel.

15. The system of claim 9, wherein the base station is configured to insert at least one training symbol adjacent to at least one null subcarrier.

16. The system of claim 9, wherein the base station is configured to transmit transmission signals for a multi-user wireless communications system.

17. The system of claim 9, wherein the base station is configured to encode the information-bearing symbols in space and time within the two or more blocks of output symbols.

18. A method comprising: in a base station: forming two or more blocks of output symbols for orthogonal frequency division multiplexing (OFDM) transmissions over a multiple-input multiple-output (MIMO) channel; identifying, via a hopping code and based at least on a cyclic prefix parameter, different positions within the two or more blocks of output symbols, the different positions comprising a block index value, a subcarrier index value, and the cyclic prefix parameter selected to compensate for intersymbol interference (ISI) associated with the MIMO channel; inserting, using the different positions identified via the hopping code, a first set of two or more training symbols and two or more null subcarriers (i) into a first block of two or more blocks of information-bearing symbols and (ii) at a first position within the first block of information-bearing symbols; inserting, using the different positions identified via the hopping code, a second set of two or more training symbols and two or more null subcarriers (i) into a second block of the two or more blocks of information-bearing symbols and (ii) at a second position within the second block of information-bearing symbols, wherein the hopping code directs the first position to be different from the second position; and transmitting over the MIMO channel, via two or more antennas, transmission signals in accordance with the two or more blocks of output symbols.

19. The method of claim 18, wherein the block index value is a positive integer.

20. The method of claim 19, wherein the subcarrier index value is a positive integer.

21. The method of claim 20, wherein the cyclic prefix parameter is a positive integer.

22. The method of claim 18, wherein the first block of the two or more blocks of output symbols and the second block of the two or more blocks of output symbols are consecutive.

23. The method of claim 18, wherein the first block of the two or more blocks of output symbols and the second block of the two or more blocks of output symbols each comprises a vector of size 1xN, where N is a total number of subcarriers in a block of output symbols.

24. The method of claim 18, wherein inserting the training symbols into the first block of the two or more blocks of information-bearing symbols further comprises inserting the training symbols into the first block of the two or more blocks of information-bearing symbols such that there is at least one information-bearing symbol between each of the training symbols in the first block of the two or more blocks of information-bearing symbols.

25. The method of claim 18, wherein inserting the training symbols into the first block further comprises inserting the training symbols into the first and second blocks of the two or more blocks of information-bearing symbols such that there is at least one information-bearing symbol between each of the training symbols in the first and second blocks of the two or more blocks of information-bearing symbols.
26. The method of claim 18, wherein each block of output symbols contains a number of training symbols equal to a number of transmit antennas.

27. The method of claim 18, wherein inserting the training symbols into the first and second blocks of the two or more blocks of information-bearing symbols further comprises inserting a number of training symbols, wherein the number of training symbols inserted per block of information-bearing symbols is based at least on a coherence time of the MIMO channel.

28. The method of claim 27, further comprising increasing the number of training symbols inserted per block of information-bearing symbols based at least on a reduction in the coherence time of the MIMO channel.

29. The method of claim 18, further comprising space-time encoding the information-bearing symbols in the first and second blocks of information-bearing symbols.

30. The method of claim 18, wherein forming the two or more blocks of output symbols, identifying the different positions, and inserting the training symbols and the null subcarriers are repeated for each of two or more transmit antennas.